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DESIGNING ALUMINUM ALLOY MEMBERS FOR COMBINED END LOAD AND BENDING

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DESIGNING ALUMINUM ALLOY MEMBERS FOR COMBINED END LOAD AND BENDING

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I. Introduction

It is not uncommon in structural design to encounter a member that is required to function as a column and as a beam at the same time. It is commonly known that the presence of the one kind of loading will decrease the capacity of the member for the other kind of loading. In designing such a member, therefore, the allowable bending moment and end load should be reduced from the values that would be permissible if either occurred alone.

The first requisite in the design of a "beam-column" is to be able to determine the strength of the member separately as a column and as a beam. The strength of the member under the combined loading can then be determined from an appropriate "interaction" equation.

This paper describes the methods of design for aluminum alloy columns, beams and "beam-columns" evolved at the research laboratories of Alcoa, following years of research in this field. The design methods herein described are being incorporated in reference 1.

Although these design procedures were devised specifically for aluminum alloys, the general principles involved should be equally applicable to other metals. There is, however, a fundamental difference in the approach adopted herein and the usual design specifications. The problem has been to derive design procedures applicable to a large number of different aluminum alloys, each in a number of different tempers, having widely differing mechanical properties and being used in a wide range of fields of application. The procedures are therefore expressed in a form suitable for determining the strength of a member, in terms of the mechanical properties of the material of the member. They are not expressed in terms of allowable design stresses. The factor of safety to be applied in a specific design will be influenced to a large extent by the circumstances peculiar to the particular problem.

From general design methods such as those described in this paper, design specifications of the usual type (in terms of allowable stresses) can be derived for a specific material in a particular field of application. This has, in fact, been done for two of the aluminum alloys most commonly used in civil engineering type structures.(2)*

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II. Strength of Columns

The primary type of failure for a member under an axial compressive load (applied at the centroids of the end sections) involves bending of the member in the weakest direction.* If the maximum load a column is capable of supporting, without buckling sideways, does not introduce stresses in the member above the elastic limit of the material, the ultimate strength is given by the Euler formula

$$f_C = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \qquad [1]$$

where f_C = ultimate strength, kips per sq. in.

E = modulus of elasticity, kips per sq. in.

r = least radius of gyration of the cross section, in.

L = length of the column, in.

K = end fixity factor.

The value of the fixity factor K will depend on the extent to which the ends of the column are restrained from moving laterally and tipping in the direction of least stiffness. In selecting a value of K for a particular problem, it is sometimes helpful to visualize "KL" as the effective length of the column, i.e., the length between points of contra-flexure when the column is in the deflected position attained just prior to failure. If the column is a member in a framed structure such as a truss, the choice of K value should be influenced by the nature of the loading in the adjacent members as well as by their size and the stiffness of the connections.

If the ultimate strength calculated by Eq. [1] is above the elastic limit of the material, the equation will overestimate the strength of the column. Various methods have been used for calculating the strength of columns in the plastic stress region. A straight line formula for column strength of aluminum alloys in the plastic range was proposed in ARL Tech. Paper No. 1(3) and was used in earlier editions of Alcoa Structural Handbook(4). While this straight line formula gives a good approximation of the column strength of the aluminum alloys for which it was derived, it was found to be less satisfactory for other alloys, particularly the artificially aged alloys such as 14S-T6, 61S-T6 and 75S-T6(5) which are currently the most important alloys involved in structural design.

As was pointed out in ARL Tech. Paper No. 1(3) and as has been confirmed by numerous additional tests, and also reported by others, equation [1] gives a good approximation of the utlimate strength of an axially loaded column in the plastic stress range if the elastic modulus "E" is replaced by the "tangent" modulus. The tangent modulus for a given stress value is simply the slope of the stress-strain curve at that stress. This "tangent modulus method" for determining column strengths in the plastic stress range is used in the aircraft industry(6) and has also been used in specifications for aluminum alloys(2).

It is a relatively simple matter to construct a column strength curve by applying the tangent modulus method to a specific stress-strain curve. It

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*Primary failure of axially loaded columns by twisting or combined twisting and bending can sometimes occur in torsionally weak members. Such failures are not common and treatment of this type of failure is considered beyond the scope of this paper.

becomes quite cumbersome, however, to apply the method to a large number of aluminum alloys, and tempers of the same, ranging in yield strength from about 4000 psi to over 70,000 psi, and having marked differences in the shapes of their stress-strain diagrams. It was decided, therefore, to seek a generally applicable straight line formula which would fit the data for all aluminum alloys better than that previously used.

Figure 1 shows some typical column strength curves and test results for aluminum alloys. It can be seen that the straight line of the type originally developed in ARL Technical Paper No. 1[3] does not give very satisfactory agreement with the test results for 14S-T6 alloy columns shown in the Figure. Applicability of a straight line of this type is limited by the requirement of tangency to the Euler curve, which means that there could be only one independent variable for different alloys, and this was taken as the compressive strength for $\frac{KL}{r} = 0$. This value was expressed as a function of the compressive yield strength of the material.[3] There was no provision for accommodating various shapes of stress-strain curves. By removing the restriction of tangency, a second variable is introduced (slope of the line) and the shape of the stress-strain curve, as well as the yield strength of the material can be accommodated.

It has been demonstrated that for materials having stress-strain curves of the type encountered in aluminum alloys, the relation between stress and strain in the region of plastic behavior of interest in determining column strengths can be expressed by a mathematical equation involving three parameters.[7] These parameters can be the elastic modulus value, the compressive yield strength value (which is the stress corresponding to a permanent strain of 0.2 per cent) and the ratio of the yield strength to a stress value corresponding to 0.1 per cent permanent strain.[8]

From an examination of the results of column tests on aluminum alloys of many different compositions and tempers, it has been determined that a straight line having the equation

$$f_C = B - D\frac{KL}{r} \qquad [2]$$

gives agreement with test results about as good as that achieved by the tangent modulus column curve when the constants are defined as follows:

$$B = S_2\left[1 + \frac{\sqrt{10S_2}}{6}\left(\frac{S_2}{S_1} - 1\right)\right] \qquad [2a]$$

and

$$D = 0.5\sqrt{\frac{B^3}{E}\left(\frac{S_2}{S_1} - 1\right)} \qquad [2b]$$

where S_2 = stress at 0.2% permanent set, kips per sq. in. (yield strength)
S_1 = stress at 0.1% permanent set, kips per sq. in.*
E = modulus of elasticity, kips per sq. in.

In order to avoid straight lines that do not meet the Euler curve, the ratio $\frac{S_2}{S_1}$ is taken to be 1.06 in these equations if the actual ratio for the material in question exceeds this value, as it seldom does for aluminum alloys. When the ratio $\frac{S_2}{S_1}$ is 1.06, the straight line of Eq. [2] is tangent to the Euler curve.

*British "Proof Stress".

If the ratio $\frac{S_2}{S_1}$ were equal to unity (a condition not encountered in aluminum alloys) Eq. [2] would give a horizontal line at a value of f_C equal to the yield strength. The straight lines that represent equation [2] in Fig. 1 were based on values of the ratio $\frac{S_2}{S_1}$ of 1.060 for 3S-H18 alloy and 1.042 for 14S-T6 alloy columns.

In reference 1, values of the constants B and D of Eq. [2] will be listed with the mechanical properties for over 200 classifications of alloy, temper and product, for wrought products, and for more than 50 classifications of castings.

III. Strength of Beams

Most aluminum alloy structural members carrying loads in bending are subject to failure by lateral buckling, although frequently only after the stresses have reached the plastic range. Some beams of compact section, however, are not subject to this type of failure. The problem of determining the strength of such beams involves relating the stress in the member to the strength of the material.

For flanged members such as I beams, channels and plate girders, in which the resistance to bending is concentrated in the material in the flanges, the ultimate bending strength can be determined by substituting the tensile strength of the material for "f" in the equation

$$f = \frac{Mc}{I}, \text{ in which} \qquad [3]$$

f = extreme fiber stress, kips per sq. in.
M = applied bending moment, in.-kips
c = distance from the neutral axis to the extreme fiber, in.
I = moment of inertia of the cross section about its neutral axis, $in.^4$.

For beams of compact section, which have considerable material near the neutral axis, the ultimate strength may be appreciably greater than that indicated by substituting the tensile strength of the material in the flexure formula [Eq. 3]. Numerous tests on aluminum alloy beams have indicated that the maximum bending moment that such members can carry can be approximated by assuming that at failure the stress over the whole cross section, both tension and compression, has reached a value equal to the tensile strength of the material. On this basis form factors can be calculated for various cross-sections. The ultimate strength of the beam can then be computed from the flexure formula [Eq. 3] by substituting for "f" the tensile strength multiplied by the form factor for the cross section. The form factor for a solid rectangular section is 1.5 and for a solid round section 1.7.

For I's, C's and Z's with equal flanges, that fail by lateral buckling within the elastic stress range,*

$$f_B = N \frac{\sqrt{EI_yGJ}}{L} \left(\frac{c}{I_x}\right) \qquad [4]$$

where f_B = critical bending stress, kips per sq. in.
E = modulus of elasticity, kips per sq. in.
I_y = moment of inertia of beam about axis parallel to web, $in.^4$.

*Only cases of bending in one of the principal planes are considered in this paper.

G = modulus of elasticity in shear, kips per sq. in.
J = torsion constant, in.^{4}
c = distance from neutral axis to extreme fiber on compression flange, in.
I_x = moment of inertia about axis normal to web, in.^{4}
N = coefficient depending on the nature and location of the applied load, the conditions of end restraint of the unsupported length "L", and the relative importance of restraint against cross sectional warping in the total resistance of the beam to twisting. (Lateral buckling involves both lateral bending and twisting).

Probably the best loading condition to assume for general design purposes is a uniform bending moment over the unsupported length "L": This is a good approximation to loading conditions frequently encountered. Lateral support is generally provided at points of load application. Eq. [4] for the case of uniform bending can be written

$$f_B = \pi \frac{\sqrt{EI_yGJ}}{KL} \sqrt{1 + \frac{\pi^2}{(KL)^2} \frac{EC_s}{GJ}} \left(\frac{c}{I_x}\right) \qquad [4a]$$

where K = factor representing lateral restraint at the end of the laterally unsupported length "L"—as for columns.
C_s = torsion-bending constant of the cross section, in.^{6}
and other terms are as previously defined.

The torsion-bending constant C_s is the property of the cross-section that measures its resistance to warping out of a plane when the member is twisted, in a manner similar to that in which the moment of inertia determines the bending stiffness.(9) Formulas are available for calculating this constant for common structural cross-sections.(10) Reference 1 will include tabulations of this value along with the other common section elements for a large number of structural shapes.

If the ends of the laterally unsupported length of a simple beam are "fixed" [K in Eq. [4a] = 0.5], Eq. [4a] gives a satisfactory approximation of the critical stress (at the center of the unsupported length) for distributed or concentrated loads applied at the neutral axis. If the ends of the laterally unsupported length are simply supported (K = 1.0), Eq. [4a] gives results that are conservative for such loading conditions on simple beams. If more precise calculation is desired for the buckling strength of beams under loading conditions other than uniform bending, use can be made of coefficients that are available for a number of different loading conditions.(11)

The critical bending stress for a beam will be influenced by the location of the load relative to the neutral axis. Loading at the top flange (without lateral restraint) will result in buckling at a bending stress lower than that for loads applied at the neutral axis. Conversely, the critical stress is increased by bottom flange loading. The effect of top or bottom flange loading can be approximated with reasonable accuracy by introducing the following multiplier terms into the appropriate equation for loading at the neutral axis.

$$f_{BB} = (f_B) \left(1 + \frac{2}{1.8 + \frac{KL}{a}}\right) \qquad [5]$$

$$f_{BT} = \left(\frac{f_B}{1 + \frac{2}{1.8 + \frac{KL}{a}}}\right) \qquad [5a]$$

where f_B = critical bending stress for loading at neutral axis, kips per sq. in. [Eq. 4a]

f_{BB} = critical bending stress when same loading is applied at bottom flange, kips per sq. in.

f_{BT} = critical bending stress when same loading is applied at top flange, kips per sq. in.

$a = \sqrt{EC_S/GJ}$

Other terms are as previously defined.

For I-beams with unequal flanges, the critical stress under uniform bending can be expressed(12)

$$f_B = \frac{\pi^2 EI_y}{(KL)^2}\left[e + \sqrt{e^2 + \frac{GJ(KL)^2}{\pi^2 EI_y}\left(1 + \frac{\pi^2}{(KL)^2}\frac{EC_S}{GJ}\right)}\right]\left(\frac{c}{I_x}\right) \qquad [6]$$

where e = distance from centroid to shear center of cross section, in. (positive if shear center lies between centroid and compression flange, otherwise negative)

Other terms are as previously defined.

Equations [4], [5], and [6] are applicable for buckling within the elastic range of the material. In many instances, however, buckling does not occur until after the plastic range has been reached. To handle plastic buckling in a simple, conservative manner, use is made of the "equivalent slenderness ratio" method.(4) The equivalent slenderness ratio is simply the slenderness ratio of a column that would buckle at the same stress as the beam in question. By equating the right hand side of Eq. [1] to the right hand side of the appropriate equation for the critical stress for a beam, one may solve for the radius of gyration "r". This value of "r" is called an "equivalent" radius of gyration" and is used with the proper effective length "KL" to form an "equivalent slenderness ratio." The critical stress for buckling of the beam, in the plastic stress range, may then be obtained by substituting this equivalent slenderness ratio in Eq. [2]. The equivalent radius of gyration for Eq. [4a] is given by the equation

$$r_e^2 = \sqrt{I_y C_S\left(1 + \frac{(KL)^2}{\pi^2}\frac{GJ}{EC_S}\right)}\left(\frac{c}{I_x}\right) \qquad [7]$$

This "equivalent slenderness ratio" method for solving the problem of the lateral buckling of beams in the plastic stress range, while not theoretically correct, gives results that are conservative, but sufficiently accurate for design purposes.

IV. Local Buckling

In the foregoing discussions relating to the strength of columns and beams, it was assumed that failure would be of a general and not a local nature. There are, however, cases in which failure occurs by, or is initiated by, "local buckling". It is considered beyond the scope of this paper to discuss the different kinds of local buckling that can influence the strength of a beam-column. Treatment of this subject can be found in many standard references(10,13) and methods of design for aluminum alloy members will be found in references 1 and 2. Buckling in the plastic stress range in reference 1 is again handled simply and conservatively by the equivalent slenderness ratio method. It is well to remember that in some cases local buckling does not result in immediate and complete failure of the member.

V. Strength of Beam-Columns

In an early treatment of the design of aluminum alloy members under combined end load and bending,[4] it was considered necessary to use two formulas, since it was recognized that failure could occur in either of two ways, i.e., by bending in the plane of the applied bending moment or by lateral buckling. Since that time a number of experimental investigations have been conducted. The results of three of these investigations, which were undertaken to study the problem of the lateral buckling of beam-columns, have been published.[14,15] A fourth investigation has explored the behavior of beam-columns that fail by bending in the plane of the applied moment.[16] The results of all of these investigations, together with some tests on eccentrically loaded tubular columns previously conducted, have led to the recommendation of one simple equation, presented below, to cover the design of beam-columns, regardless of the type of failures.

For problems involving combined loading, it is convenient to express the strength of a member in the form of an "interaction equation" incorporating "stress ratios".[17] The simplest such equation for combined end load and bending is of the form

$$\frac{f_c}{f_C} + \frac{f_b}{f_B} = 1 \qquad [8]$$

where f_c = applied end load, kips per sq. in.

f_C = strength of member under end load alone, kips per sq. in.

f_b = applied bending stress, kips per sq. in. ($f_b = \frac{M_b c}{I}$)

f_B = strength of member under bending alone, kips per sq. in. ($f_B = \frac{M_B c}{I}$)

This equation has been shown to be inadequate for the design of aluminum alloy beam-columns.[14,15,16] The equation recommended for this purpose, based on the above mentioned investigations, is

$$\frac{f_c}{f_C} + \frac{f_b}{f_B(1 - f_c/f_{CE})} = 1 \qquad [9]$$

where f_{CE} = the Euler critical load, Eq. [1], for buckling of the member in the direction of the applied bending moment, kips per sq. in.

Other terms are previously defined.

The use of such a simple equation to handle a complex problem entails some sacrifice in accuracy. It is thought, however, that Eq. [9] provides a reasonably accurate and sufficiently conservative basis for the design of aluminum alloy-beam columns. The extent to which the equation is supported by the test results can be seen in Figs. 2 and 3.

Figure 2 shows the results of all the tests in which failure was by lateral buckling. Tests were on I section members in which bending was applied in the plane of the web. Bending moment was introduced by applying the end loads eccentrically. The ratios of column strengths of the specimens in the plane of, and normal to the plane of, the web ranged from about 9 to 1 to about 0.9 to 1. Failure involved lateral buckling in both the elastic and plastic stress range.

In Figure 3 have been plotted the results of tests in which the members failed by bending in the plane of the applied moment. The specimens were

of four kinds, (1) H-section, with bending in the plane of the web; (2) solid rectangular section, with bending in the weak direction; (3) hollow rectangular section (representative of members with material concentrated in flanges) with bending in the weak direction; and (4) hollow circular sections. The test results for the type 4 specimens, which have not been previously published, are summarized in Table I.

VII. Comparison of Interaction Equations

Various interaction equations have been proposed or used for the design of members under combined end load and bending. It is not contended that all of these equations have a rational theoretical basis; in fact it is not required that an interaction equation be so based in order to be satisfactory. All that is required is that the equation provide a simple and reasonably accurate means for calculating the strength of such a member. In explaining the differences between the various equations, however, it is informative to consider the meaning of the relationships expressed by the equations.

The simple interaction relationship expressed by Eq. 8 is the basis for the treatment of the beam-column problem in a design specification for steel structures.(18) The interaction relationship expressed by Eq. 8 has the following meaning: Failure occurs when the sum of the applied stresses ($f_c + f_b$) reaches a limiting value that varies from f_C (when $f_b = 0$) to f_B (when $f_c = 0$), the variation being linear with the ratio $\frac{f_c}{f_C}$ or $\frac{f_b}{f_B}$. That this equation is not in reasonable agreement with the results of tests on aluminum alloy beam-columns is shown in Fig. 4. The strength indicated by Eq. 8 ranges from 16% below to 39% above the test results. In general, the use of this equation to predict the strength of the beam-columns tested would be decidedly unconservative.

The interaction equation proposed in this paper [Eq. 9], differs in meaning from the relationship expressed by Eq. 8 in this respect—the significant stress is considered to be not ($f_c + f_b$) but $\left[f_c + f_b\left(\frac{1}{1 - f_c/f_{CE}}\right)\right]$. The multiplier applied to f_b is an approximate correction to take account of the fact that the bending stress at the center of the member is greater than the applied bending stress f_b because of the additional bending moment resulting from the end load and deflection of the member. The reasonably conservative agreement between Eq. 9 and the results of tests on aluminum alloy beam-columns has been shown in Figs. 2 and 3. In Fig. 5, all the test results, regardless of type of failure, have been plotted in comparison with this proposed interaction equation. The equation predicts strengths ranging from 18% below to 10% above the test results.

As previously stated, the original treatment of the design of aluminum alloy beam-columns in Ref. 4 included two equations to take account of the two general types of failure. Expressed in interaction form and in the terminology of this paper, these equations can be written

$$\frac{f_c}{f_C} + \frac{f_b}{f_B\,(1 - f_c/f_B)} = 1 \qquad [10]$$

and

$$\frac{f_c}{f_C} + \left(\frac{f_b}{f_B}\right)^2 = 1 \qquad [11]$$

Equation [10] was intended to cover failure by bending in the plane of the applied moment, and Eq. [11] failure by lateral buckling.

Equation [10] was derived(19) by assuming that at failure the sum of the direct stress and bending stress is equal to f_B, and defining the maximum bending stress as $f_b\left(\frac{1}{1 - f_c/f_C}\right)$. In Fig. 6 this equation is shown in comparison with the test results from the aluminum alloy beam-columns that failed by instability in the plane of the applied moment. It can be seen by comparing this plot with Fig. 2 that Eq. 10 is definitely inferior to Eq. [9] as a reasonable representation of the behavior of a beam-column. Equation [10] predicts strengths ranging from 10% above to 31% below the test results. This equation will be unconservative in the region of small $\frac{f_b}{f_B}$ values for long members, since the actual maximum stress at failure may be considerably lower than f_B. For short members the equation may become ultra-conservative because use of the factor $\left(1 - \frac{f_c}{f_C}\right)$ rather than $\left(1 - \frac{f_c}{f_{CE}}\right)$ overcorrects for the effect of deflection, since f_C may be appreciably lower than f_{CE}.

Eq. [11] was based on Timoshenko's theoretical solution for the lateral buckling of beams under axial load and equal bending moments applied at the ends.(13) The inadequacy of this equation to represent satisfactorily the strength of such members is shown in Fig. 7. This subject has been discussed in a previous publication,(15) in which it is shown that equation [9] can be derived from a modification of the Goodier theoretical solution(9) for lateral buckling failure by making certain simplifying assumptions.

VIII. Conclusion

When combined with the methods outlined herein for calculating the strength of aluminum alloy columns and beams, the interaction relationship expressed by Eq. [9] gives a reasonably accurate, simple means for calculating the strength of members subjected simultaneously to a compressive end load and bending. This conclusion is based on tests involving equal end moments. Additional investigational work is required to determine the limits of applicability of this interaction equation under such conditions as unequal end moments, end moments of opposite sign, and transverse loads, with varying degrees of end restraint.

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TABLE I

RESULTS OF TESTS ON ECCENTRICALLY LOADED COLUMN SPECIMENS OF ALUMINUM ALLOY 17S-T4 ROUND TUBES, 2 IN. O. D. BY 0.083 IN. WALL*

Specimen No.	Slenderness Ratio L/r	Eccentricity In.	Average Compressive Stress at Failure, kips per sq. in.
6-102	150	0.024	4.39
12-102	150	0.141	4.17
13-102	150	0.263	3.82
14-102	150	0.377	3.60
5-54B	80	0.017	15.06
10-54A	80	0.137	12.51
10-54B	80	0.255	10.88
11-54A	80	0.435	9.78
4-27B	40	0.018	28.93
4-27C	40	0.117	24.80
4-27D	40	0.240	21.58
9-27A	40	0.427	18.18

*Data taken from unpublished report of tests conducted at Aluminum Research Laboratories by Marshall Holt. Specimens tested on platens with ball-bearing spherical seats with the centers of rotation in the bearing surfaces of the platens.

Average compressive yield strength (set = 0.2%) determined on specimens of the full cross section was 38.53 kips per sq. in.

Average modulus of failure in bending (maximum bending moment divided by section modulus) was 64.0 kips per sq. in.(21)

Values of compressive strength for zero eccentricity, determined by plotting test values of critical axial load vs the corresponding critical bending moments and extending curves through these points to the axis of zero bending moment, were 4.5, 15.9, and 30.1 kips per sq. in. for specimens with slenderness ratios of 150, 80, and 40, respectively.

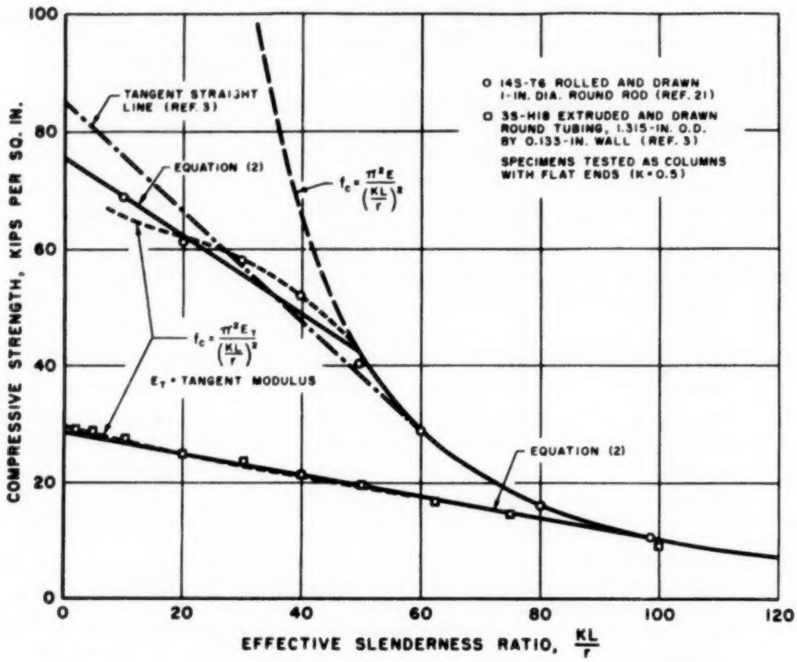
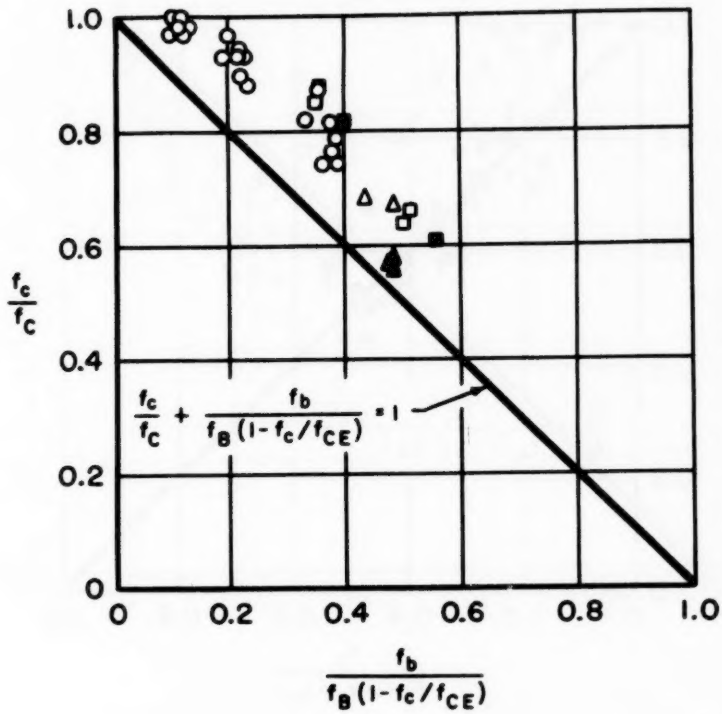


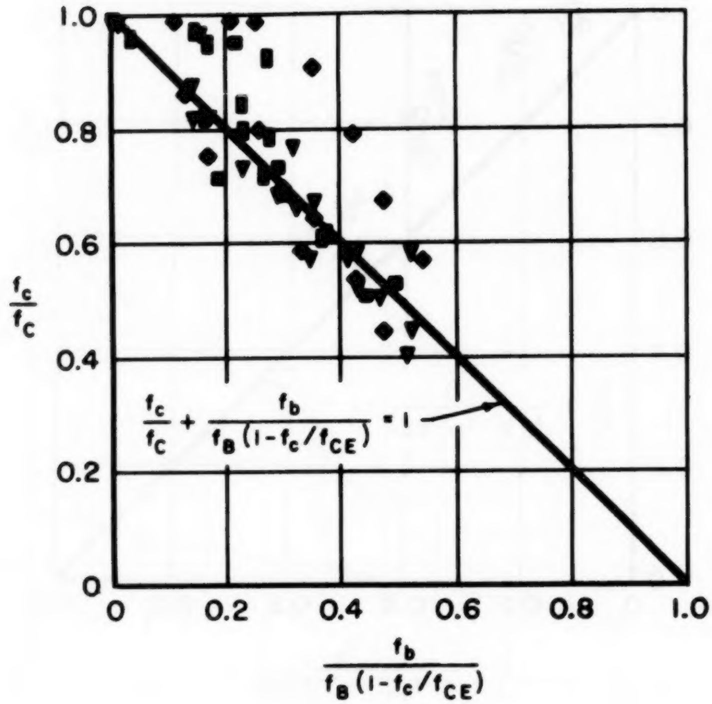
FIG. 1 – STRENGTH OF ALUMINUM ALLOY COLUMNS



○ 27S-T6 I-SECTION (REF. 15)
△ 14S-T6 I-SECTION (REF. 15)
□ 14S-T6 H-SECTION (REF. 15)

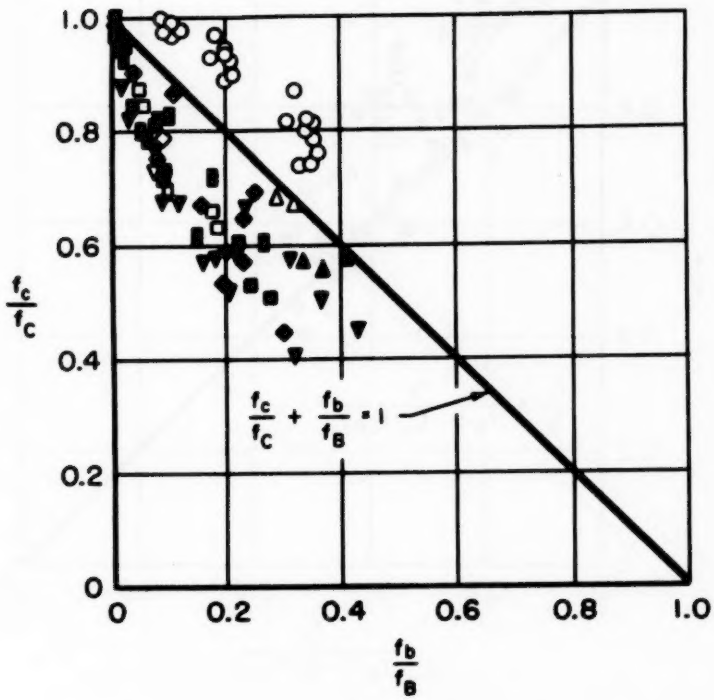
NOTE:- SOLID SYMBOLS INDICATE PLASTIC FAILURE

FIG. 2 - COMPARISON OF TEST RESULTS WITH INTERACTION EQUATION (9) - FAILURE BY LATERAL BUCKLING



■ 14S-T6 H-SECTION (REF. 15)
▼ 61S-T6 RECTANGULAR BAR (REF. 16)
◆ 61S-T6 RECTANGULAR TUBE (REF. 16)
▮ 17S-T4 ROUND TUBE (TABLE 1)

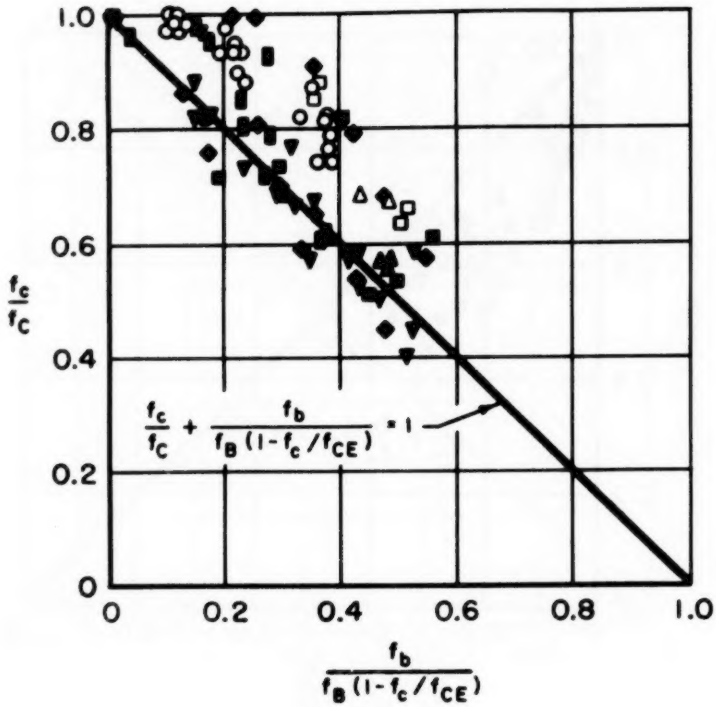
FIG. 3 - COMPARISON OF TEST RESULTS WITH INTERACTION EQUATION (9) - FAILURE BY BUCKLING IN PLANE OF APPLIED BENDING MOMENT



○ 27S-T6 I-SECTION (REF. 15)
△ 14S-T6 I-SECTION (REF. 15)
□ 14S-T6 H-SECTION (REF. 15)
▽ 61S-T6 RECTANGULAR BAR (REF. 16)
◇ 61S-T6 RECTANGULAR TUBE (REF. 16)
▯ 17S-T4 ROUND TUBE (TABLE I)

NOTE:- SOLID SYMBOLS INDICATE PLASTIC FAILURE

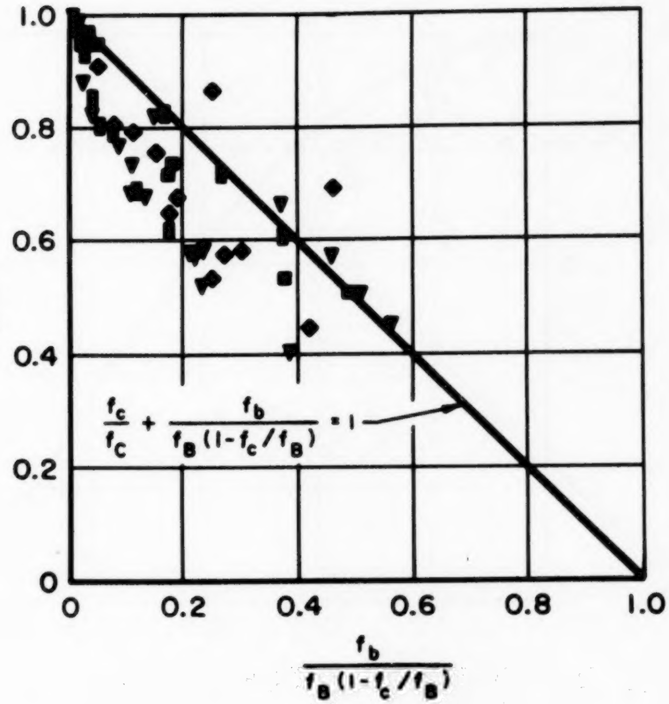
FIG. 4 - INTERACTION DIAGRAM FOR ALUMINUM ALLOY BEAM-COLUMNS, EQUATION (8)- BOTH TYPES OF FAILURE



○ 27S-T6 I-SECTION (REF. 15)
△ 14S-T6 I-SECTION (REF. 15)
□ 14S-T6 H-SECTION (REF. 15)
▽ 61S-T6 RECTANGULAR BAR (REF. 16)
◇ 61S-T6 RECTANGULAR TUBE (REF. 16)
▯ 17S-T4 ROUND TUBE (TABLE I)

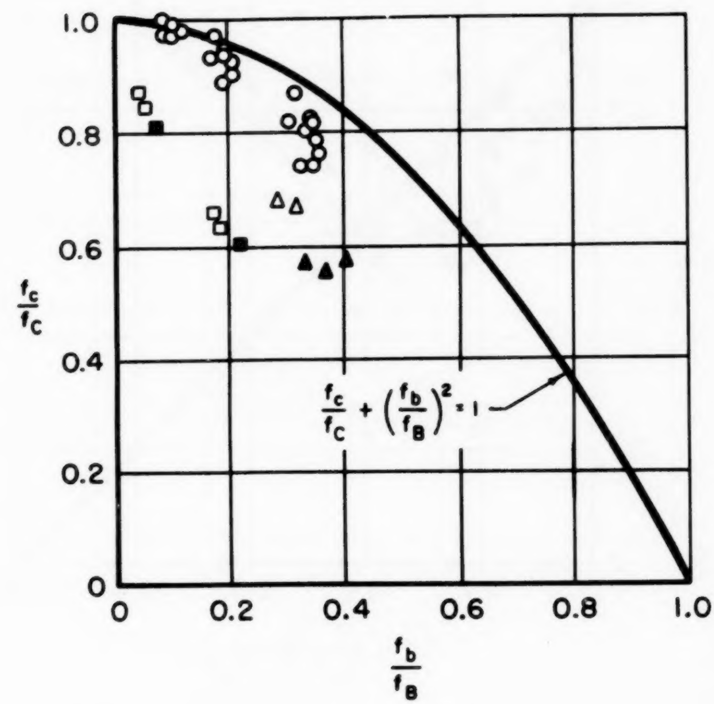
NOTE:- SOLID SYMBOLS INDICATE PLASTIC FAILURE

FIG. 5 - INTERACTION DIAGRAM FOR ALUMINUM ALLOY BEAM-COLUMNS, EQUATION (9)- BOTH TYPES OF FAILURE



■ 14S-T6 H-SECTION (REF. 15)
▼ 61S-T6 RECTANGULAR BAR (REF. 16)
◆ 61S-T6 RECTANGULAR TUBE (REF. 16)
▮ 17S-T4 ROUND TUBE (TABLE I)

FIG. 6 – COMPARISON OF TEST RESULTS WITH INTERACTION EQUATION (10) – FAILURE BY BUCKLING IN PLANE OF APPLIED BENDING MOMENT



○ 27S-T6 I-SECTION (REF. 15)
△ 14S-T6 I-SECTION (REF. 15)
□ 14S-T6 H-SECTION (REF. 15)

NOTE :- SOLID SYMBOLS INDICATE PLASTIC FAILURE

FIG. 7 - COMPARISON OF TEST RESULTS WITH INTERACTION EQUATION (11) - FAILURE BY LATERAL BUCKLING